Master's degree thesis

LOG950 Logistics

Inventory under return on investment maximization

Anna Gribkovskaya

Number of pages including this page: 53

Molde, January 2012

Molde University College

Mandatory statement

Each student is responsible for complying with rules and regulations that relate to examinations and to academic work in general. The purpose of the mandatory statement is to make students aware of their responsibility and the consequences of cheating. Failure to complete the statement does not excuse students from their responsibility.

<table>
<tr><td colspan="3">Please complete the mandatory statement by placing a mark <u>in each box</u> for statements 1-6 below.</td></tr>
<tr><td>1.</td><td>I/we herby declare that my/our paper/assignment is my/our own work, and that I/we have not used other sources or received other help than is mentioned in the paper/assignment.</td><td>☒</td></tr>
<tr><td>2.</td><td>I/we herby declare that this paper
1. Has not been used in any other exam at another department/university/university college
2. Is not referring to the work of others without acknowledgement
3. Is not referring to my/our previous work without acknowledgement
4. Has acknowledged all sources of literature in the text and in the list of references
5. Is not a copy, duplicate or transcript of other work</td><td>Mark each box:
1. ☒
2. ☒
3. ☒
4. ☒
5. ☒</td></tr>
<tr><td>3.</td><td>I am/we are aware that any breach of the above will be considered as cheating, and may result in annulment of the examinaion and exclusion from all universities and university colleges in Norway for up to one year, according to the <u>Act relating to Norwegian Universities and University Colleges, section 4-7 and 4-8</u> and <u>Examination regulations</u> section 14 and 15.</td><td>☒</td></tr>
<tr><td>4.</td><td>I am/we are aware that all papers/assignments may be checked for plagiarism by a software assisted plagiarism check</td><td>☒</td></tr>
<tr><td>5.</td><td>I am/we are aware that Molde University college will handle all cases of suspected cheating according to prevailing guidelines.</td><td>☒</td></tr>
<tr><td>6.</td><td>I/we are aware of the University College`s <u>rules and regulation for using sources</u></td><td>☒</td></tr>
</table>

Publication agreement

ECTS credits: 30

Supervisor: Øyvind Halskau

Agreement on electronic publication of master thesis

Author(s) have copyright to the thesis, including the exclusive right to publish the document (The Copyright Act §2).
All theses fulfilling the requirements will be registered and published in Brage HiM, with the approval of the author(s).
Theses with a confidentiality agreement will not be published.

I/we hereby give Molde University College the right to, free of charge, make the thesis available for electronic publication: ☒**yes** ☐**no**

Is there an agreement of confidentiality? ☐**yes** ☒**no**
(A supplementary confidentiality agreement must be filled in)
- If yes: **Can the thesis be online published when the period of confidentiality is expired?** ☐**yes** ☐**no**

Date: January 2012

Preface

This master thesis is a cumulative result of my experience and skills I obtained from studying Logistics at Molde University College. It was interesting and challenging process and I really satisfied with the results I get so far. This master program gives me good and various knowledge, help to develop new skills and personal qualities.

First and foremost I would like to thank my supervisor associate professor Øyvind Halskau for his opportune response and very useful comments on my progress.

I also want to thank all my teachers at Molde University College for their qualified assistance during my studies. I'm also very grateful to all my friends and co-students for their cooperation and discussions we have on the topic of my thesis.

Finally, I want to thank my family for their support and belief.

Abstract

Inventory control is a vivid and crucial question for the majority of the companies. There are a lot of deterministic models and solutions for different types of businesses and inventories. This thesis attempts to move from the most common cost minimizing model towards the return on investments maximizing one and after this to compare them. In the first part named “Introduction” the most important concepts are defined and different types and models of inventory control are introduced. In the second part named “Literature and historical overview” the short description of the previous researches made on the considered topic are introduced. Also, some scientific discussions and their outcomes are observed. In the third part the problem is defined, formulated and solved. Also formulas for some general parameters and variables are presented and discussed a bit. In the fourth part, the results obtained in the previous part are carefully investigated and discussed. In the fifth part a numerical example is introduced in order to clarify the theoretical results and makes it more obvious. In the last section a general outcomes are discussed and the possibilities for further researches are presented.

Table of contents

1. Introduction and some general concepts

Most of the researchers working in the field of operational management or supply chain management consider the inventory control as one of the most crucial questions for the companies. Every organization (banks, schools, shops, producers and manufacturers) in the economy sector has items of some kind to be kept as inventories and has to deal with these inventories (Waters, 2003). Inventory is considered as one of the most expensive assets for majority of the companies, dealing with finished goods, spare parts or raw materials. Financial analysts use inventory to asset ratio - amount of assets that are ties up in inventory, in order to judge about the inventory cost. For many firms this ratio can be very high, up to 40% (Silver et al, 1998). It can be useful to calculate how much the firm spend for the inventory compared to the sales revenue. For example, Tersine (1994) insists that average manufacturing firm spends more than half of its sales revenue on inventory (including services, parts, components and raw materials). Nevertheless, companies need to have it. Firms need inventories even if they cost a lot due to many reasons but the most general and important is "to provide a buffer between variable and uncertain supply and demand" (Waters, 1992). The reasons for holding the inventory may vary for different groups of goods, for example raw materials, finished goods or semi-finished goods, but in general they are the following:

- In decoupling stock inventory is used as a buffer between two inter-dependent operations in order to prevent breakdowns or unevenness in production rates. In this case inventory also reduce the need for output synchronization
- To correct the mismatch between supply and demand. For example, for raw materials inventory the production plan changes in response to the market situation for finished goods (sales, orders etc). The demand varies with the production plan. Inventories allow to have the required quantity and item for production when needed. Also inventories help to deal with cyclical and seasonal demand. Market situation for many types of gods depends on

season of year (holydays, festivals and etc). Companies may stock up goods and hold inventories in order to meet the increased demand.

- To correct forecast errors. For example, if the demands are larger than expected or at unexpected times.
- To correct delivery errors. For example, if the deliveries are delayed or too small.
- To elude delays in delivery goods to the customers.
- To capitalize the price discounts on large orders
- To buy more if the price is expected to go up. For example, if it’s known that price will increase in few months the company should buy goods in advance and hold stocks to compensate the increased costs.
- To buy more if some item is expected to be out of production or is difficult to buy/deliver.
- To buy in bulk in order to make full loads and reduce the transportation costs (it also reduce the time as well, as far as transit time for full container shipment is faster unlike part shipment. In the case of part shipment one part waits for other loads to fill the container which can take several weeks)
- To provide a reserve for emergencies
- To sustain stable level of operations.

The total investments in inventories are huge, and the control of capital tied up in raw materials, work-in-progress, and finished goods gives opportunity for improvement, which can give a significant competitive advantage for the company. In general there are many surrounding circumstances in process of managing inventory. For example in order to answer the main question “How much to order?” one should first made the demand forecast, estimate the price and decide about the service lever needed for this type of goods (Waters, 1992).

1.1 Inventory control

As far as inventory is of great importance for the firms (at least some of them) one of the most important questions is "how to manage it?" Every firm is trying to make stock holdings as efficient as possible, but first of all one should decide how to measure inventory efficiency. Most common measures to judge the efficiency are:

- The amount of stock held
- The holding cost
- Number of shortages when demand cannot be met
- Frequency of the turnover
- So on.

One of the most common measure which helps to judge the performance is service level given to the customers. Service level is percentage of customer demand which can be met from directly from stock. The level of customer service depends directly on the amount stored in the inventory, but at the same time the cost of holding such stock. That's why most of the firms have a common aim to minimize the cost of inventory having some specified customer level. That is why so called cost model (the cost minimization one) is most common in the inventory management now.

There are different methods of assessing demand. According to this methods there are two fundamentally different approaches to inventory control. There are dependent and independent demand systems. ***Independent demand systems*** treat all items independently – it means there is no connection between demands for different items; all demands are independent from each other. In this case the aggregated demand for an item is made up of demands from different customers, which are also independent (Donald Waters, 1992).

This model can use either two different types of orders: the fixes quantity or periodic reviews. Systems which use ***fixed order quantity*** place an order of fixed size when inventory level fall down to certain point. Systems which use ***periodic review*** place orders of variable size but in the same time intervals.

Independent demand inventory is a theoretical approach, useful for some inventory problems, especially for managing the stocks of finished goods and spare parts. In real life the supply and demand for different items are often related, for example the demand for different parts of the product depend on the final demand for product itself. For such situations the dependent demand approach is more appropriate.

The ***dependent demand systems*** deal with cases of strongly interdependent items. The demand for such items depends on the demand for other items. The most common case of such items is the connection between the demand of the materials and the demand of finished product. That’s why such systems use production plans to forecast the demand for each item (method is formalized, for example, in MRP- “material requirements planning” and the associated concept “Bill of Material”). Independent demand systems are most useful for inventories supporting production.

As a sort conclusion one can mention that independent demand systems are more useful for finished goods and the dependent demand systems are more useful for work in progress.

My master thesis consider inventory model for independent demand systems.

1.2 Types of inventories

There are many different classifications of the inventory systems, which differ according to the used criteria. For example, if we consider information flow as such criterion, all the inventory systems can be divided into two types, such as perpetual inventory or continuous inventory and non-continuous or one-periodic inventory.

Continuous inventory is inventory system in which the information is a function of doing business. In such system the demand information is updated on a continuous basis as (Hadley and Whitin, 1963). The traditional cost minimizing model is almost always related to the perpetual inventory.

In the ***non-continuous inventory*** system updates are made on a periodic basis. One of the most common examples of periodic inventory is so-called the newsboy problem (a classical example of random demand). That is the single period inventory model. The story is the following: there is a newsboy who sells newspapers on the corner of the street. He needs to place an order the day before he will sell the newspapers. The order can be placed only once (for the same newspaper) and newspapers can be sold should be sold in one day because the papers only have any value on the day they are published. The next day they have no value. If the order is too big the newsboy will lose money of the unsold papers, and if it's too small he will lose profit and annoy customers. Getting the correct order quantity is the aim of the model. This problem is described in many textbooks and articles on the inventory management, for example in the "Inventory management and production planning and scheduling" (Silver et al, 1998). The optimal order quantity in this case should maximize the expected profit. For such problems the probability models are used for inventory control. In real life managers need to calculate the order quantities for firms, so the newsboy model may be used, for example, to solve the problem of two independent firms, a supplier and the retailer. In this case both of the firms want to maximize their own profits. The supplier has a certain unit production cost. When retailer places an order to the supplier, he faces the well known newsboy problem. The cost per unit now is the supplier's production cost and the price is some given market price per unit (obviously cost should be less than price) (Sven Axsäter, 2006). My thesis considers the inventory systems associated with continuous inventory.

1.3 Inventory models

The most common inventory model associated with continuous inventory is the EOQ model. This inventory model uses the criterion of cost minimization in order to find the optimal size of the order. Such models give an inventory-related equation (the EOQ formula) that determines the optimum order quantity that a company should hold in its inventory in order to minimize variable costs. These costs will be comprised of order or setup cost, cycle inventory holding cost, cost of safety stock and stock out cost. In general there are some basic categories of costs (Silver et al, 1998, Sven Axsäter, 2006):

- Basic production or purchased costs (unit cost). These costs are associated with goods themselves; the annual amount the firm needs buy/produce multiplied with unit purchase/production price. This cost is commonly the largest part of the total cost associated with the inventory.
- Inventory carrying costs (holding cost). By this cost one have an opportunity cost for capital tied up in the inventory. The cost of capital commonly is the biggest part of holding cost; other parts may be insurance, taxes, material handling, storage etc. Holding cost is determined as fraction of the unit value, which may vary for different types of items. In general case this fraction should exceed the interest rates in the bank. In fact there are some other costs which can be included in the inventory holding costs. For example, the cost of the capital and storage cost. These costs can be quite large, according to Meredith and Shafer (1999) for some average manufacturer it may be up to 35% of the cost of basic production.
- Ordering or set up cost. These costs are associated with each replenishment and do not depend on the order size. For example, in production, there are set up and learning costs. It also can be some administrative costs which arouse when the order is placed and some other costs associated, for example, with transportation and material handling.
- Costs of insufficient capacity in the stock run (shortage cost or service constraints). There are various costs which can appear if the item is ordered, but not delivered from stock due to shortage. It depends on customers: whether

they agree to wait while the order is backordered or not. In both cases firm will face some extra money loss. In the first case the money will be lost due to additional administrative procedures and in the other case the lost sale will reduce the net profit of the firm. The shortages may also influence the sales in the long run. Shortages can be avoided if the lead time is short.

- Control systems costs (costs associated with the operation of the decision system).
- Costs of changing work force sizes and production rates

Some of these costs can be incorporated together in a group of costs which consider acquisition and transportation of materials required for production, storage, handling, and further delivery of the items to customers. Such commonly named logistical costs. In my thesis it will be used as a parameter introduced for better understanding of the formulas of order quantity, budget constraints and shadow prices.

The traditional inventory model that is almost always related to the perpetual inventory is formulated as follows:

Minimize Total Cost

The total cost (TC) is a function of the order size and generally include the purchasing cost of all units, inventory holding cost and ordering cost. There is also a set of assumptions, associated with such model:

- A single item is considered
- All costs are known and do not vary
- Demand is known and stable
- Lead time is known and deterministic
- Goods are ordered and delivered in batches
- No stock out situations. 100% service level
- Time horizon is short (no reinvestment of returns)
- Each stock item is independent (that is do not share any recourses with other items)

There can be other assumptions of the model some of which may seem unrealistic, as far as any model is just a simplification of the reality. In practice the model gives useful results even if the assumptions never can be met in the real life. Even if the results are not optimal in the reality they can be a very good approximation. Also the model can be expanded in different ways in order to apply it to some real situations. Using such model one will obtain an optimal order size or the so called economic order quantity (EOQ), which is found to minimize the TC. This approach has a number of weaknesses, one of which is actually the number of unreal assumptions. This can be overcome by developing more complicated models. In most of the cases this models are too complicated and require a lot of time and efforts to solve. Another threat of traditional analysis is that cost and demand, which are used in calculating the EOQ, may contain errors, for example, due to wrong forecast for demand or wrong estimation of cost. Wrong data make the development of more complicated models useless, because such models will also use the same wrong date and that may lead to wrong result anyway. There some other drawbacks of using the traditional analysis, for example in production one has a high batch set up costs, that can lead to large batches and complicated production schedule, required for such batches and in the end we will have long lead times for customers, excessive storage capacity and huge amount of money tied up in the inventory. Such problems can be solved by putting some other value for holding cost in the classical model. This new holding cost should be artificially high and lead to smaller batch sizes, however this is not the optimal solution any more, just some attempt to find balance between conflicting objectives. However, despite of all criticism EOQ still can be used as a guideline to drive inventory and it also helps to have better understanding of the inventory holdings (Waters, 1992).

A short list of some other weaknesses of the EOQ model:

- It does not consider the nature of goods, for example, one may obtain the fractional value for goods that can exist only in discrete state (cars, computer systems, houses, etc).
- Suppliers often have standard package and may not be willing to split them. For example if some liquid is sold in five litter bottles it will be not possible to buy thirteen litters.

- Suppliers deliver goods using standard vehicles, having fixed capacity. If for example the capacity is less than the order size one will be double charged for the delivery, because the needed order size will require two deliveries.

In all the cases which were named above it may be better to round order quantity to a more "convenient" number. In general the best argument in advantage of EOQ analysis is that cost grows up slowly near to optimal. It allows to obtain good results using close approximation (Waters, 1992).

So far, regardless of the classification scheme used for identification of the inventory models, most formulations of treating inventory has of course an objective to maximize profit either directly or indirectly by minimizing cost (EOQ model) with price and demand exogenously determined (Johnson and Montgomery, 1974). This approach has many strong points, but there are also some weaknesses, for example, different models build on this approach, fail to recognize the interaction of an inventory with other parts of the company (so called company-wide approach). But there exists a third alternative named return on investment (ROI). According the definition by Trietsch (1995) "ROI is the ratio between profit (before tax) and owners' equity". Order quantity formula obtained by using such criterion will give the order quantity that a company should hold in its inventory to maximize the ROI. The ordering policy based on classical model considers the economic order quantity (EOQ) and gives most suitable and reliable result, but will not necessarily yield a good ROI. At least, a better ROI can be obtained by deviating from the classical EOQ order sizes. For the investor or the share holders in a firm this can be more important than getting bigger profit. There can be different cases associated with this, for example reducing capital inventory lead to lesser profit but higher ROI. First attempt in adopting the EOQ to the objective of maximizing ROI has been made in 1930s (Raymond, 1931). The order quantity, obtained by using the ROI as an objective can be called the ROQ (ROI-maximized order quantity) similar with the EOQ. In 1964 Tate stated that ROQ=EOQ, that is not actually correct in the general case. The correct relationship is ROQ≤EOQ. The reason and the consequences will be discussed below. As one will see, the ROI as an objective for the inventory control that may give us a completely different result compared to the result obtained by an EOQ model. The crucial question now is when and why should one use ROI instead of other objectives?

Pros and cons of using ROI as a performance evaluator have been discussed in many research papers and are actually well-known. For example Kaplan (1982), made a short description of advantages and disadvantages of using ROI to examine company performance. If to consider advantages, one should mention, that ROI encourages goal correspondence, as far as it can be compared the cost of capital indexes used for external reports, which do not depend on size or type of business. At the same time use of ROI encourages more efficient utilization of the assets, due to managers are motivated to invest only in case of possibility of ROI increase rather than an only profit increase can be obtained. The reason of using ROI in this case is the problem of availability of funds. The inventory management deals with opportunity cost for money tied up in the inventory. This is an assumption which can be used if there are no budget constraints. If the situation is opposite, and there are some constraints, the corresponding opportunity cost is the return on the last investment. The problem is that such investment can be identified only after the decision was made, and that's why one needs some other criterion to allocate funds. In the short-term situations one may use ROI as such criterion. This argument is actually the best one in favour of ROI, especially for small businesses where the capital budget constraint exists. This is a common situation for retailers and investment centres. In case of such upper limits fall below the capital requirements necessary to obtain the desired profit, the opportunity cost of the capital tied up in inventory is no longer fixed, as inventory control theory states.

Of course there are some disadvantages of using ROI to examine company performance. One of the most substantial shortcomings it may happen that managers will try to get more ROI level by cutting down the capital. It also may encourage managers to avoid investments which give return more than firm's cost capital, but decreases the unit's overall ROI. The possible way out of such situation is to use residual income (RI) instead of ROI (Moorse and Scneider, 1979).

The main problem in my thesis will be to explore how order sizes should be formed if company has an objective to maximize ROI for their inventory control systems and compare this with the traditional EOQ approach.

2. Literature and historical overview

"Hindsight is always 20/20"

The oldest production/inventory model is EOQ model developed in 1915 by F. W. Harris. It is usually called the Wilson formula, because he (R. H. Wilson) started to apply it extensively (Hax and Candea, 1984). Let d_i be demand for item *i*, A_i the fixed cost per order for item *i* (or in production it is the cost of setting up production), v_i unit cost for item *i* and *r is* an interest rate. The total number of items is *n*. In this case the EOQ model will give the order quantity equal to the $(Q_i)_{HW}$:

$$(Q_i)_{HW} = \sqrt{\frac{2A_i d_i}{v_i r}} \qquad i = \overline{1,n} \tag{1}$$

Based on the minimization of the TC function:

$$\mathrm{TC}(\mathrm{Q_i}) = \sum_{\mathrm{i=1}}^{\mathrm{n}} \frac{\mathrm{d_i}}{\mathrm{Q_i}} A_i + \frac{1}{2}\sum_{\mathrm{i=1}}^{\mathrm{n}} \mathrm{Q_i} v_i r; \qquad i = \overline{1,n} \tag{2}$$

Quite another answer will be derived if one changes the objective to maximize ROI. The ROI-maximization order quantity (ROQ) model has also quite long history. Originally the argument starts not in the inventory management but in production scheduling. In order to determine optimal batch-size in batch production one should have clearly define the objective. Elion introduces four different objectives: the minimization of total cost, the maximization of profit for the batch, maximum return and maximum rate of return. A few words should be mentioned here to prove that the results obtained by Eilon can be used for inventory as well. The mathematical model he used to describe cost per unit is similar to the cost function used in the inventory and the batch size obtained for minimal total cost is very much similar to the Wilson formula (with some other constants introduced, of course).

The famous argument between Tate and Eilon took place after the publication of "Economic Batch-Size Determination for Multi-Product Scheduling" (1960) and "Dragons in pursuit of the EBQ" (1964) by Elion. The response from Tate was a letter: "In defence of the economic batch quantity" (1965) published by Operational Research Society. Eilon considers the four different criteria: minimization of total cost, maximization of profit, maximization of return and maximization of rate return (return divided by time). Production optimization problem is closely connected to inventory optimization problem, due to all unsold items kept in the inventory after they are produced. The amount the production department produce is the same which stock department order on stock (and vice versa). The economic batch quantity is similar to economic order quantity due to the same criteria used for optimization. In the article in case of cost minimization the batch-size for production is the same as the EOQ formula for the inventory (just written a bit differently), the profit maximization gives quite predictable conclusion that the larger batch led to increase of the profit. The most interesting part is the maximizing of return and maximizing rate of return. Return is defined as profit divided by cost and the solution obtained after running the model gave same the result as cost minimization case. That was not strictly correct and the discussion runs forward and led to the some other relationship between this to quantities. The fourth case was the rate of return and in this case obtained result was less then cost-minimization one. In the article “Economic Batch-Size Determination for Multi-Product Scheduling” Eilon state that the maximization of profit gives the same result as the minimization of cost per unit, thus yield to the same optimal batch-size.

Almost all literature published in the area of inventory management consider profit or cost as an appropriate criterion for choosing the optimal inventory policies. That is the case of using EOQ model. At the same time there is an interesting question “How much to invest in order to get the needed profits?” which is not discussed in such models. As far as it is possible to get the same profit given that the investments are different, the model based on profits alone will not treat both situations equivalent. Considering the return on investments one will get different inventory decision rules than minimum cost or maximum revenue models may offer.

However, the ROQ model is more mathematically challenging and not that common as EOQ. Maximization of ROI as a criterion for the inventory stands in contrast to the EOQ model which maximize profit or minimize cost. One of the first articles which is devoted to the ROQ model was the article of Schroeder and Krishan (1976) in which authors make an attempt to develop the model utilizing the ROI concept and introduce the name ROQ similar to the traditional EOQ model. They introduce the formulation of lot size model which was based on the return on inventory investment (ROII). One of the reasons why it was necessary to introduce the new model can be explained from the point of view of how to measure the company performance. For example, Chamberlain states:

"Two companies may show the same return on sales, but if one requires twice as much investment to achieve the result, it would be stretching a point to claim that their performances were equally good" (Chamberlain, 1962).

The assumptions made in the model are those associated with the Wilson EOQ model (Schroeder and Krishan, 1976).

The article of Schroeder and Krishan also contains the discussion of ROI as a criterion. For example, its sphere of application: the kinds of inventory problems for which the ROQ model will be suitable. It is obvious that it does not suite the nonprofit organizations such as schools, hospitals and some governmental organizations. Another interesting point is the usage of the model according to the type of goods. Authors state that ROQ is most appropriate for the firms that deal with finished goods, such as retailers and wholesalers, because their assets can be inventories themselves. Quite different situation occurs if the firm has raw material or in-process inventories, such inventories are held to forward finished goods and not considered as investments themselves. For this reason the raw material or in-process inventories may be better controlled using classical cost minimization model. Summarizing all together one may say that in the profit-making firms the ROI concept can be used to control inventories for finished goods. This lead to different decision rules (from that of cost minimization model) which is more appropriate when inventory is viewed from the shareholders' or owner's point of view, as far as owners are always interesting in the maximization of return on the investment. There was another attempt to move from the EOQ model, made by Moorse and Scneider in 1979. As opposed to Schroeder and Krishan they were using residual income (RI) as an objective. Below there will be some discussion on this topic.

Arcelus and Srinivasan continue in 1985 with the model of ROI-maximized EOQ under the variable demand and markup rates. They introduce the price inventory models based on ROII (which was provided by Schroeder and Krishan in 1976) and include constant price elasticity. The idea was the following: "if inventories are to be what they are, namely assets, inventory policy should be more sensitive to demand fluctuations and should use the pricing mechanism as well as the ordering policy to optimize performance indices, such as return on investment". In the paper the demand was assumed to be a function of price while the price, unit cost and markup on unit cost was assumed to be constant (or subject to learning effects). The inventory policy is considered in its relationship to the marketing and accounting. The retail industry was taken as an example, of the industry were this interaction is most obvious. In the retail industry the inventories are considered and evaluated as any other asset, according to their profit-generating capability. Before, while using the marketing-inventory interface in finding the optimal marketing strategy one should consider first the price that maximize the revenue for known demand curve and after this the resulting demand and price should be used as parameters for EOQ model. All the models at that time treat price and demand as parameters or try to move the classical TC minimization criteria in order to reach such objectives as ROI. Arcelus and Srinivasan made an attempt to combine marketing, inventory and accounting objectives and consider them simultaneously. In order to reach this goal the authors extend the traditional EOQ formulation and include price as a decision variable as well as order quantities and used ROI as an objective. Actually ROI was used instead of profitability index (PI). PI is present value of benefits per dollar invested. Using ROI in this case is allowed since for the case of short-term investment allocation on the basis of ROI is the same as allocation on the basis of profitability index. This paper makes good overview how ROI can be used in case of incorporation of the inventory policy in other subsystems of the company. The most interesting outcomes were that in the used model neither price, demand nor cost was treated as constant parameters. Also this model gave a huge possibility of extensions.

Two years after this article, in 1987 Arcelus and Srinivasan published a new one on the same topic, making an attempt to extend the deterministic EOQ model to reflect different optimizing criteria. The main purpose this time was to develop the decision rules for management of finished goods inventories. They address their paper to the retailing and other businesses where inventories are treated not on the traditional least cost base, but as a profit-generating asset. So they continue to treat inventory as an asset, but the goal now was to develop a pricing policy in order to obtain the required level of demand which in it’s turn will optimize the objective. It was obvious that traditional cost-minimization model doesn’t suit this case, as far as the demand and price in such model are externally determined and could not be influenced by the firm.

Almost at the same time a new concept named “zero inventory” (ZI) appeared in the literature. ZI is a kind of philosophy that argues against the EOQ model in order to reduce the inventory to its lowest possible level. The objective is to reduce a set up cost. What is a set up cost in this case? If one consider the classical EOQ model it contains a parameter named ordering cost or the cost of set up production. This is actually the set up cost in the case of inventory since inventory itself needed to avoid the additional frequency of placing orders or setting up the production. There are some consequences of EOQ which can be incorrect or lead to incorrect decisions. They are:

- If set up cost is reduced this will reduce the level of inventory
- Since EOQ models give robust formulas for the optimal value, reducing the set up costs will not have any significant influence on the total cost
- Reducing the set up cost will lead to reduction of TC (both production and inventory)

Actually all statements are true under the assumptions made in the formulation of EOQ model, but in more general production systems they may be false. There are many other possibilities actually, for example the reduction of set up cost may lead to increase of TC and increase of inventory lever as well (Willard Zangwill, 1987).

ZI was introduced as a way to deal with a list of weaknesses which may applier while using the EOQ approach. They are: uncertainty in supply and demand, bad quality, poor scheduling and the most important the set up cost. ZI is almost synonyms with “Just-In-Time” (JIT) production and “Japanese Inventory”. These terms describe a kind of philosophy based on elimination of every action or investment which do not create value directly. There are many articles and research papers which consider different cases of application this kind of philosophy and also a lot of attempts to create a mathematical model which will describe how to move from traditional EOQ model to the ZI. In one of such research considers the influence which such philosophy make on the returns. It turned out that applying the ZI creates increasing marginal returns. This is a kind of inverse task to the one that was considered under the objective of ROI maximization. It was found that order quantity obtained in the case of ROI maximization is less than the same obtained using the EOQ formula. So the EOQ and ZI philosophy gives some extreme cases of guiding inventory, while using the ROI as a criterion allows to find the reduced order quantity (that suits the JIT objective of inventory level reduction) and at the same time to have a robust solution for the given inputs. So, the ROQ may be a good “middle stop” in very common nowadays attempt of moving from EOQ to ZI.

The ROI as objectives was considered by the researches as a tool to incorporate the price theory and inventory control, or for example inventory control and various demand models. There were very few researches which consider ROI itself. Most of the papers ROI model is considered together with other models in order to provide comparative analysis for all of them and also to go further from the classical inventory EOQ model which assumes the demand stable and exogenously determined. For example, Rosenberg in1991 made an attempt to decide between profit and ROI as a criterion for the inventory in case of the monopoly firm. He developed optimal solutions to the economic theory of the firm (ETF), profit and return on the inventory investments (ROII) models. The demand was not a parameter any more; it was a function of price. The mathematics inside the models was a bit complicated and actually this may be the reason why there were no deeper researches on the topic. At least, the non-linear demand seems to be interesting to consider. If we go from the strict assumptions and consider the demand forecast, we will face the problem of using the unknown demand in the model, classically assuming the demand estimated and stable. This question was solved in the EOQ model by using the exponential smoothing for demand (see for example Sven Axsäter, 2006), but I fail to find other articles considering this question in ROQ model. Of course, this question goes beyond my thesis, but it may be a possibility for further researches. There also was an interesting numerical example by Rosenberg (1991), which clearly shows the difference between ROI and profit as an inventory objective. According to his data for the same inputs different models gives different values of profit and return, for example the profit in ETF and profit maximizing model was almost the equal, at the same time ROI model gives just half of this profit. But when it comes to the return, the ROI model shows almost 3.5 times more return than the ETF and profit maximizing model, which have almost the same.

In the Dan Trietsch article (1995) the ROQ model is considered as EOQ for company-wide ROI maximization. It gives some traces in the past explaining the reason why the EOQ formula became that much common in contrast to the ROQ. Trietsch state that: "*For a profitable firm, if one ignores the non-inventory investment, then specify smaller orders than EOQ (and thus reducing the average inventory) will increase the return on the inventory investment (even if total costs increase)*". Unfortunately, when Tate in 1964 was considering the problem of need to take in the account the total investment in the inventory model he concluded that it is not true and come out with the solution that in general case the EOQ is equal to ROQ. Starting from this point all further inventory models use the EOQ approach as a most significant (and the most simple). Some of his opponents, for example Eilon find many arguments against EOQ criteria (Eilon, 1964; Tate, 1965). In order to promote short lead times managers take into considerations the penalties of having large inventory such as holding costs, but Trietsch state that this practice implies the use of models such as EOQ or ROQ. He shows why the Tate's conclusion was false and come out with the correct relationship between EOQ and ROQ that is ROQ≤EOQ. He also shows that ROQ grows by less than the square root of the demand (there will be discussed below that in other approaches the demand is not present in the ROQ formula at all). There are conditions (discussed by Trietsch) under which the ROQ is not a function of demand. He states that only items whose demand changes during the lifecycle still require the EOQ model to drive the inventory. There is one more problem, connected with ROQ model, which was discussed in the article. In order to have an accurate solution one needs to decide how the ROI function should look like. For example, the definition for the ROI used in this thesis is not the same as the one used by Trietsch. But there is something they have in common. For example, Trietsch ignores taxes, due to the assumption that ROI before taxes strongly correlated with ROI after taxes.

There are some other papers considering the application of the ROQ model in different business situations. For example, some of them consider the investment in the set up operations and quality improvement. Otake et al. tried to construct and analyze inventory and the investment in set up operations under ROI maximization (1999). In the article they make an assumption that set up cost is a rational or linear function on level of investment. They show how the inventory level can be reduced when it is rational to invest in setup operations. They also state that the motivation for the paper was the lack of mathematical models with ROI as an economic performance criterion when it is an option of investing in the set up operations. The most interesting points in the paper are the formulation of the ROI maximization problem and the reduction of the inventory level in such investments. In the other article of Otake and Min (2001) the main problem is investment in quality under the same conditions (ROI-maximization). In the paper ROI is established as the characterization of the global optimal solution was given. These two articles use the opportunity which was introduced in the Schroeder and Krishan paper when they discuss the ROI as a criterion. In the published literature on the inventory control the profits and costs are quite popular to discuss, but at the same time there is a lack of discussion about how much should be invested (when it is a possibility to invest) and how it will influence the inventory policy.

This master thesis is partly based on the research made by Halskau and Thorstenson in their paper “The EOQ and the ROQ and Their Relations to the ROI” (1998).

3. The Order Quantity and other parameters for ROI Maximization

3.1 Short problem description

The problem in the case is to find significant way to manage the inventory when the objective is to maximize the ROI. As it was mentioned using ROI as an objective for inventory control have its advantages, especially if one consider short-term cases and a company-wide approach. The ROI as a criterion may be also used, for example, if the firm has many shareholders and as far as dividends are paid as a percentage of the gross profit shareholders are interested in the maximizing ROI more than in slashing the total costs of the firm. There is a list of assumptions, mostly the same as that for the EOQ model; besides this it is one more strict assumption: there is no money tied up apart from the inventory. That means that there is no set up investments in the infrastructure or some other capital investments, but the goods itself. This is quite common situation for the vendor managed inventory (the case when supplier takes full responsibility for maintaining inventory, according the information provided by buyer) and/or third part logistics. In this situation the set up costs for business are quite small, but the goods itself and the other costs connected with inventory may be high (ordering cost, rent and so on).

3.2 ROQ vs EOQ

In the classical and the simplest approach of the inventory theory for the family of items can be formulated as done in (1):

Minimize $$\mathrm{TC}(\mathrm{Q_i}) = \sum_{i=1}^{n} \frac{\mathrm{d_i}}{\mathrm{Q_i}} A_i + \frac{1}{2}\sum_{i=1}^{n} \mathrm{Q_i} v_i r; \qquad i = \overline{1,n} \tag{2}$$

where

TC(Q_i) = total cost for item *i*

Q_i = order quantity for item *i*

d_i= demand for item *i*

A_i = the fixed cost per order for item *i*

v_i = unit cost for item *i*

n = number of items

r = interest rate

The order quantity obtained by such method is the well-known Harris-Wilson formula (1) which is broadly used in the inventory theory.

$$(Q_i)_{hw} = \sqrt{\frac{2A_i d_i}{v_i r}} \tag{1}$$

However simple examples shows that these order sizes do not give the best ROI, by reducing (1), better ROI can be obtained. However, oppositely to the EOQ case that allow to treat two or more commodities independently of each other, the ROI model cannot treat them independently. That can make the model quite complicated. There is also a possibility when ROQ is equal to EOQ. This is possible if inventory is not a part of equity (Trietsch, 1995), so in the thesis the other situation is considered. The items on stock are purchased by capital invested in the firm permanently.

3.3 Problem solution

Let's consider a firm that uses the ROQ approach for the inventory policy. The investments in this case can differ in a quite broad sense – from the fixed costs of set up the business to the specific investments in improving the quality and etc. In the most simple case (which is quite good example in order to show how the method can be applied) the firm uses subcontracting for their set up operations and there is no other investments apart the inventory. In this case the problem can be formulated as follows:

Task: Find the ROI-maximized order quantity which is optimal for the firm under the condition of no investments tied up apart from the goods.

In order to solve this problem we will consider the ROI function which depends on some parameter, which can be introduced as a capital limitation in the traditional optimization problem with budget constraint:

$$\sum_{\mathrm{i=1}}^{\mathrm{n}} \frac{\mathrm{d_i}}{\mathrm{Q_i}} A_i + \frac{1}{2} \sum_{\mathrm{i=1}}^{\mathrm{n}} \mathrm{Q_i} v_i r; \qquad i = \overline{1, n} \tag{3}$$

Subject to

$$\frac{1}{2} \sum_{\mathrm{i=1}}^{\mathrm{n}} \mathrm{Q_i} v_i \leq B; \tag{4}$$

where

B = budget constraint

Applying the Lagrange method for the optimization problem to the model described in (3) and (4), the following Lagrange function will be obtained (G. M. Fikhtengol'ts, 1965):

$$F = \sum_{i}^{n} \frac{d_i}{Q_i} A_i + \frac{1}{2} \sum_{i}^{n} Q_i \, v_i r + \lambda \left(\frac{1}{2} \sum_{i}^{n} Q_i v_i - B \right) \tag{5}$$

The extreme values in this case can be found by taking partial derivation of the Lagrange function for Q_i.

$$\frac{\partial L(Q_i,\lambda)}{\partial Q_i} = -\frac{Ad_i}{Q^2{}_i} + \frac{1}{2}v_i r + \frac{1}{2}\lambda v_i \tag{6}$$

$$\frac{\partial L(Q_i,\lambda)}{\partial \lambda} = \frac{1}{2}\sum_{i=1}^{n} Q_i v_i - B \tag{7}$$

By solving these equations we will get the explicit formulas for both Q_i and λ:

$$\lambda = \frac{1}{4B^2}\left(\sum_{i=1}^{n}\sqrt{2A_i d_i v_i}\right)^2 - r \tag{8}$$

$$(Q_i)_B = \sqrt{\frac{A_i d_i}{v_i}}\frac{2B}{\sum_{i=1}^{n}\sqrt{A_i d_i v_i}} \tag{9}$$

One of the interesting outcomes in this case is that the value of Q_i does not depend on the ordering cost if it is equal for all types of goods. For example if ordering cost include just some administrative costs which arouse when the order is placed and do not contain other costs associated, for example, connected with transportation and material handling (or they are the same for all items). In this case we may subtract the ordering cost from the formula and get it even simpler:

$$(Q_i)_B = \sqrt{\frac{d_i}{v_i}}\frac{2B}{\sum_{i=1}^{n}\sqrt{d_i v_i}} \tag{11}$$

This is quite significant difference from the order quantity obtained from the EOQ model. In the case of EOQ the order quantity doesn't change in case of equal ordering costs for different items (the situation is quite common, especially for third part logistics, were the transportation cost may be the same for all items). In order to compare the obtained order quantity with the EOQ one the formula for Q_i can be rewritten as follows:

$$(Q_i)_B = \frac{B\sqrt{2r}}{\sum_{i=1}^{n}\sqrt{A_i d_i v_i}}(Q_i)_{HW} \tag{12}$$

In this formula we have a budget constraint, which is not estimated yet. In order to find an exact solution for the problem of maximizing ROI we should find a budget constraint and the order quantity corresponding to this budget. According to the definition of the *B*, given by (4), we may introduce one more budget constraint, the one corresponding to the Harris-Wilson order quantity:

$$\frac{1}{2}\sum_{i=1}^{n} v_i (Q_i)_{HW} = B_{HW} \tag{13}$$

After this we can rewrite formulas for Q_i and λ, according to this new constraint

$$(Q_i)_B = \frac{B}{B_{HW}}(Q_i)_{HW} \tag{14}$$

$$\lambda = \left[\left(\frac{B_{HW}}{B}\right)^2 - 1\right] r \tag{15}$$

The economical meaning of *λ* can be explained as a “cost” of invested money. Formally in the constraint optimization the value of Lagrange multiplier named a shadow price. If we go back to the description of the Lagrange method and the Lagrange function, we may see that the Lagrange function looks almost the same as the cost function, the only difference is this part containing *λ*. If we consider it more precisely we may mention that *λ* is a kind of coefficient which shows how costly the bad utilization of money may be. For instance, there are three possible situations with the part containing *λ*:

- It may be equal to zero. In this case $\frac{1}{2}\sum_{i=1}^{n} Q_i v_i = B$ and we have the classical TC which doesn’t contain any additional “cost” for the capital.
- It may be more than zero. In this case $\frac{1}{2}\sum_{i=1}^{n} Q_i v_i > B$ and we have the additional “cost” for the capital, which was utilized over the budget. This can be easily understood. For example we have a certain budget constraint, but have used more money or need to use more. In this case we should pay for the money we spend beyond the budget (bank interest, opportunity cost of using this money in some other project, etc). The value of *λ* in the case will be a percentage of the money we pay for the every extra dollar used. We may in some way call it a “marginal penalty” for budget deficit.

- It may be less than zero. In this case $\frac{1}{2}\sum_{i=1}^{n} Q_i v_i < B$ and we have the additional "value" for the capital, which should be subtracted from the TC. For example we have a certain budget constraint, but have used less money. In this case we may save this money for some other projects (and have the opportunity cost subtracted from the TC).

It will be more discussion on the topic after we get the explicit formula for this parameter, which will not contain any other unknown values, such as budget constraint, but only the known constants such as ordering cost, price and demand.

Going back to the problem formulation it is obvious that in order to obtain the solution it's needed to use some proper mathematical method that will give the ROQ as an extreme point of ROI function under the maximum condition. In the article of Halskau and Thorstenson (1998) the calculus was used. In the case of EOQ two or more commodities can be treated independently of each other, the ROI model cannot treat them independently.

As a general concept the ROI can be described as the net profit divided by the capital employed in the investment project:

$$ROI = \frac{g(Q)}{I(Q)} \tag{16}$$

where

g(Q) = the gross profit,

I(Q) = the total capital employed.

ROI as a function of the order size can be formulated as follows:

$$ROI(Q_i) = \frac{\sum_{i=1}^{n}(p_i - v_i)d_i - \Phi - \sum_{i=1}^{n}\frac{d_i}{Q_i}A_i - \frac{1}{2}\sum_{i=1}^{n}Q_i v_i r}{\frac{1}{2}\sum_{i=1}^{n}Q_i v_i + L} \quad i = \overline{1,n} \tag{17}$$

where

Φ = is fixed cost per time unit which is independent from the demand,

L = is capital employed, but not tied up in the inventory.

Further, authors come out with the extensions of the model introducing L - the capital employed but not tied up in the inventory. As far as the problem formulated above considers no investments tied up apart from the goods, we should either use the same model stating that the value L is equal to zero or use some other way to solve the problem.

One of the alternatives is to use the Lagrange method of multipliers in order to find the extreme value of the function. In order to find the optimal solution some kind of boundary equation can be used as an introduction of budget constraint.

The objective function (17) is a function of order cost. It can be easy rewritten as a function of other variable, in this case it is a budget constraint (for full description see Appendix I). This is done by applying

$$ROI((Q_i)_B) = \frac{\sum_{i=1}^{n}(p_i - v_i)d_i - \Phi - (\sum_{i=1}^{n} \frac{A_i d_i}{\frac{B}{B_{HW}}(Q_i)_{HW}} + \frac{1}{2}\sum_{i=1}^{n} \frac{B}{B_{HW}}(Q_i)_{HW} v_i r)}{\frac{1}{2}B} \quad (18)$$

As it easy to see the only variable in (18) that is unknown is B. By applying just a standard method of finding the extreme value of the function one may find the explicit formula for maximum budget that can be spend on inventory in order to get the maximum ROI. By setting to zero the first derivative of (18) with respect to B we will get the value of B_{max}:

$$B_{\max} = \frac{2\sum_{i=1}^{n} \frac{A_i d_i B_{HW}}{(Q_i)_{HW}}}{\sum_{i=1}^{n}(p_i - v_i)d_i - \Phi} \quad (19)$$

In order to prove that this budget constraint is the extreme maximum of the ROI function, we need to consider the second derivative of (18) at the point B_{max}, that should be (and actually is) less than zero (for full description see Appendix II).

The formula can be reformulated using the new parameter: one is net profit before the logistical costs taken away (*H*) and another one total logistical costs which one will obtain using the Harris-Wilson order sizes (TC_{HW}).

$$H = \sum_{i=1}^{n} (p_i - v_i) d_i - \Phi \tag{20}$$

After introducing *H*, we will have the following formula for budget constraint:

$$B_{\max} = \frac{2rB^2{}_{HW}}{H} \tag{21}$$

Now we need to get the TC_{HW}. This can be done after some reformulation of Harris-Wilson budget constraint (see Appendix III).

$$TC_{HW} = \sum_{i=1}^{n} \sqrt{2A_i d_i v_i r} \tag{22}$$

Hence, we will have another formula for B_{max}:

$$B_{\max} = \frac{1}{2rH} TC^2{}_{HW} \tag{23}$$

Now, if we go back to Lagrange multiplier and rewrite it according to new variables, we will obtain:

$$\lambda = \left[\left(\frac{H}{TC_{HW}} \right)^2 - 1 \right] r \tag{24}$$

Now we can find the formula for the order quantity in case of ROI maximizing:

$$(Q_i)_{B_{\max}} = 2\sqrt{\frac{A_i d_i}{v_i r}} \frac{\sum_{i=1}^{n} \sqrt{A_i d_i v_i}}{\sum_{i=1}^{n} (p_i - v_i) d_i - \Phi} \tag{25}$$

It may be also written using the new parameters *H* and TC_{HW}:

$$(Q_i)_{B_{\max}} = \frac{1}{r} \sqrt{\frac{2A_i d_i}{v_i}} \frac{TC_{HW}}{H} \tag{26}$$

This formula doesn't contain any parameters, which should be estimated additionally. At the same time, we may see that items are not treated independently any more, even if we considered the case when the demand of one item doesn't influence the demand for the rest of them, the order quantity consist for one item depends on the parameters associated with other items. This will be discussed more precisely while analyzing the results.

After obtaining the formula for order size we can determine the cost function corresponding to new approach. It can be made by using (26) in (2):

$$TC_{ROQ} = \frac{rH^2 + TC^2{}_{HW}}{2\sqrt{r}H} \tag{27}$$

TC function consists of ordering cost (OC) and inventory holding cost (HC):

$$OC_{ROQ} = \frac{\sqrt{r}H}{2} \tag{28}$$

$$HC_{ROQ} = \frac{TC^2{}_{HW}}{2\sqrt{r}H} \tag{29}$$

One can mention that the ordering cost in general is not equal to the inventory holding cost, as it is in the case of EOQ model.

4. Analysis of the results

4.1 The budget constraint

The budget constraint is quite an interesting parameter which actually can be introduced in the Harris-Wilson model as well, but in the ROQ model it has a crucial role. The budget constraint, according to definition is the amount of money the firm is allowed to spend on inventory (in our case, at least). There exists some research for the ROI used as an inventory objective, but there was almost nothing said about budget limitations (the possibility was mentioned, but no actual formulas or calculations made). So the formula obtained for this constrained worth some discussion. We have calculated at least three formulas (19, 21, 23) in order to find the budget constraint. Let's consider them more precisely. From the formula described in (19) we may see that the case when $\sum_{i=1}^{n}(p_i - v_i)d_i = \Phi$ is strictly forbidden (as it is forbidden to divide by zero). This is quite obvious because in this case firm makes no profit and we may not use the ROI as an objective (it can be used only for profitable organizations). At the same time it is obvious that such approach will make sense only if $\sum_{i=1}^{n}(p_i - v_i)d_i \geq \Phi$, so the net profit is positive. Otherwise the optimization process will not give any valid results. When it comes to parameters, such as price, demand and order cost for every item it seems a bit hard to see the influence of each of them, as we have the summation of all items in most of the formulas. Of course we may also consider a single item problem, but this may not be correct in the general case, as far as ROI criterion is appropriate for a company-wide approach. At least we can say something about the behavior of the function. For example, if price for one unit goes up, the budget constraint should increase as well. This is quite obvious, even if the price for this particular unit is small compare to all the rest, the nominator should increase and denominator should decrease, that definitely will lead to the whole formula will increase. It may be quite interesting to look at the budget constraint when it's written using new parameters H and TC_{HW}:

$$B_{\max} = \frac{1}{2rH} TC^2{}_{HW} \tag{23}$$

As we may see, the limitation that should be used for the budget is in directly proportional to the square of total logistical costs which one will obtain using the Harris-Wilson, and inversely to interest rate and net profit which do not include the logistical costs. This is quite useful information for people in the firm who are making financial forecasts and control inventory. Budget constraint may help them have better insights into the problem. If the business environment changes fast and there is no accurate information about it, knowledge about some average rules may help to make a decision that will correspond the chosen objective. The budget constraint may be a good tool for decision-making in this case.

4.2 Shadow price of the invested capital

The formal definition of the shadow price is the value of the Lagrange multiplier appeared in the optimal solution. This value means that it is the extremely small change in the objective function is caused by an extremely small change in the constraint. This can be easily proved if we look the gradient of the objective function at the optimal solution. As it is known from the mathematical analysis (Fikhtengol'ts, 1965) the gradient in this case will be a linear combination of the gradients corresponding to the constraint functions with weights equal to Lagrange multipliers. In the considered problem we have only one constraint, so the Lagrange multiplier is a shadow price for this constraint. As far as our constraint was setting up the limitation on money, we may say that in this case the Lagrange multiplier is a shadow price of the invested capital.

Let's consider this value more precisely, because the value of the shadow price can give us a powerful tool for decision-making and help to have better insights into considered problems. As it was already mentioned the value of *λ*, was described as "cost" of capital utilization. The fist formulation we have for *λ* was given in (15):

$$\lambda = \left[\left(\frac{B_{HW}}{B} \right)^2 - 1 \right] r \qquad (15)$$

As it was already mentioned the order quantity obtained in the case of ROI maximization should be less or equal to the Harris-Wilson order quantity that means that the budget constraint in this case should be also less or equal to the budget constraint obtained using the EOQ. According to this logic $\frac{B_{HW}}{B} \geq 1$, that is quite obvious, because it would be strange is firm reduce their TC by using more money for the inventory. In the case when B_{HW} is close to B, the money firm would lose using EOQ model will not cost a lot, but the more the difference between B_{HW} and B will be, the larger the multiplier before r will become. For example, if the $\frac{B_{HW}}{B} = 1{,}2$, we will obtain the value of $\lambda = 0{,}44r$, but for $\frac{B_{HW}}{B} = 1{,}5$ we will get $\lambda = 1{,}25r$. In this example ratio of the budget constraints changes just for 25%, but the value of *λ* increase in almost three times (2,84). After we get the explicit formulation for the maximum budget constraint and introduce some new parameters, we obtain a new formulation for λ (24):

$$\lambda = \left[\left(\frac{H}{TC_{HW}} \right)^2 - 1 \right] r \tag{24}$$

This formula actually looks the same as the previous one. Nevertheless it may be more convenient to use, due to the budget constraints are in some way auxiliary parameters. The net profit and the total logistical costs are commonly used by supply chain managers and other people in the firm dealing with inventory as a quantitative measure of the inventory efficiency. The other point is that they are quite easy to calculate and forecast.

4.3 Order quantity

In the optimization process the explicit formula for order quantity in the case of ROI maximization was obtained (25):

$$(Q_i)_{B_{\max}} = 2\sqrt{\frac{A_i d_i}{v_i r}} \frac{\sum_{i=1}^{n} \sqrt{A_i d_i v_i}}{\sum_{i=1}^{n} (p_i - v_i) d_i - \Phi} \tag{25}$$

As it was already said the order quantity in the model that uses ROI maximization instead of cost minimization may be less or equal to the classical EOQ. Furthermore, there are some important properties of the order size, which can be derived just from the view of the function:

- If the cost *Φ* will increase, the net profit *H* will decrease, as *Φ* is included in *H* with the minus sign. The *H* stands in the denominator and due to this all order quantities will increase.
- If demand d_i is increasing or decreasing that is quite difficult decide how the order size will change as far as it is included both in numerator and denominator. At the same time demand rate cannot be subtracted from the formula, so the order quantities should change in some way.
- If the ordering cost A_i will increase in case of any item, the order quantities for all of the items will increase.
- If the interest rate *r* will increase the order sizes will decrease for all items.
- If the selling price p_i for any of the items will increase, then the net profit *H* will increase and the denominator will increase. All order quantities will decrease in this way.
- If the purchasing cost v_i for any item *i* will increase all order quantities for all items except item *i* will decrease. In the case of item *i* it is not obvious in which way the order size will change.

4.4 Cost function

4.4.1. Total cost

The TC function obtained using the EOQ model may be formulated as follows:

$$TC_{HW} = \sum_{i=1}^{n} \sqrt{2A_i d_i v_i r} \qquad (22)$$

As a result of optimization process the following formula for TC_{ROQ} was obtained:

$$TC_{ROQ} = \frac{rH^2 + TC^2{}_{HW}}{2\sqrt{r}H} \qquad (27)$$

Formula (27) for TC_{ROQ} gives a relation between cost function in case of EOQ model and the new cost function in the ROQ model. Let's consider some outcomes from this formulation:

- If the cost Φ will increase, the net profit H will decrease, as Φ is included in H with the minus sign. The H stands in the denominator and in the numerator, but the nominator dominates the denominator and that's why the TC_{ROQ} in this case will decrease (Actually functions of the type f(x) =x+1/x are increasing for all x≥1, so we should assume that H is positive and big enough).
- If demand d_i will increase, the net profit H will increase, as d_i is included in H and the TC_{HW} will increase as well. In this case nominator dominates the denominator and the TC_{ROQ} in this case will increase.
- If the ordering cost A_i will increase in case of any item, the TC_{ROQ} will increase.
- If the interest rate r will increase the TC_{ROQ} will also increase.
- If the selling price for any of the items will increase, then the net profit H will increase. The H stands in the denominator and in the numerator, but the nominator dominates the denominator and that's why the TC_{ROQ} in this case will increase.
- If the purchasing cost for any item i will increase it is not obvious in which way the TC_{ROQ} will change.

4.4.2. Order cost

TC function contains the ordering cost (OC):

$$OC_{ROQ} = \frac{\sqrt{r}H}{2} \tag{28}$$

As one can mention the main difference between OC in EOQ model and the ROQ model is that in traditional model OC depends on the parameter A_i, but in the model under consideration it's not. Below there are some outcomes from this formula:

- If the cost Φ will increase, the net profit H will decrease and OC_{ROQ} will also decrease.
- If demand d_i will increase, the net profit H will increase and OC_{ROQ} will also increase.
- If the interest rate r will increase the OC_{ROQ} will also increase.
- If the selling price for any of the items will increase, then the net profit H will increase and OC_{ROQ} will also increase.
- If the purchasing cost for any item i will increase the net profit H will decrease and OC_{ROQ} will also decrease.

4.4.3. Inventory holding cost

TC function contains the inventory holding cost (HC):

$$HC_{ROQ} = \frac{TC^2{}_{HW}}{2\sqrt{r}H} \tag{29}$$

There are some outcomes from this formula:

- If the cost Φ will increase, the net profit H will decrease and HC_{ROQ} will also increase, as far as H stands in denominator.
- If demand d_i will increase, the net profit H will increase and HC_{ROQ} will decrease, as far as H stands in denominator.
- If the interest rate r will increase the HC_{ROQ} will also increase, because r stands in the denominator and in the numerator, but the nominator dominates the denominator.
- If the selling price for any of the items will increase, then the net profit H will increase and HC_{ROQ} will decrease.
- If the purchasing cost for any item i will increase the net profit H will decrease and HC_{ROQ} will increase.

5. Numerical example

Let's consider a family of six different items which has the same ordering cost A (just to make calculations a bit simple) and vary a bit in demand and purchasing values. The table below shows the input parameters such as price {p1,p2,..,p6}, purchasing value {v1,v2,..,v6} and demand {d1,d2,..,d6}. We will assume the A=200 $, cost Φ=27 000$ and interest rate r=10%.

d1	500	v1	25$	p1	35$
d2	350	v2	150$	p2	200$
d3	400	v3	130$	p3	170$
d4	800	v4	50$	p4	70$
d5	470	v5	80$	p5	100$
d6	620	v6	75$	p6	100$

Table 1: Inputs for the numerical example: data for price, demand and purchasing value of the items

Let's find the Harris-Wilson order quantities for such case.

EOQ1	EOQ2	EOQ3	EOQ4	EOQ5	EOQ6
283	97	111	253	153	182

Table 2: Order quantities in the case if Harris-Wilson model is used

After this is done, we will try to move from this solution to the ROI-maximizing one. For this we will consider the budget constraint, changing from B_{HW} to B_{max}. In order to do this we will consider the ratio B_{HW}/B={1.1;1.2;1.3;..B_{HW}/B_{max}}. The value of B_{HW}/B_{max} predicted by the model is 7.03. The ROQ quantities corresponding to such ratio are:

ROQ1	ROQ2	ROQ3	ROQ4	ROQ5	ROQ6
40	14	16	36	22	26

Table 3: Order quantities in the case if ROQ model is used

On the figure below we may see the graphic illustration of the process of improving the ROI by changing the budget constraint. As we may see from the graph the maximal value of ROI is obtained when B_{HW}/B=7. It's quite close to the result we obtain theoretically; the small difference can be explained by the size of the "step" we have chosen. If we chose smaller "step", for example B_{HW}/B={1.01;1.02;1.03;..B_{HW}/B_{max}}, we will be able to get more precise answer.

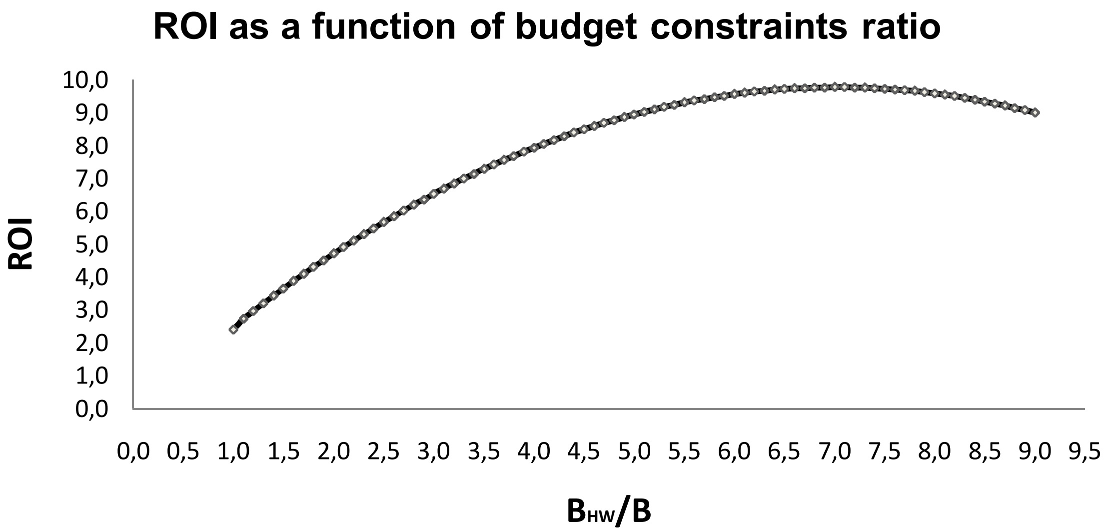

Figure 1: The graphic illustration of the dependence between ROI and the budget constrained B.

If we consider the graph more detailed we may see how the value of ROI changes while moving from one model to another. The starting point, the ratio equals to 1 is the classical EOQ model. By changing the ratio we also change the ROI until the function get to the maximum point and start to decrease. The ROI corresponding to EOR model is around 2.4 and the maximum ROI is around 9.8. Results for profit and ROI in case of both models are presented in the table below.

$Profit_{ROI}$	$Profit_{EOQ}$	ROI_{EOQ}	ROI_{ROQ}	$Profit_{ROQ}/Profit_{EOQ}$	ROI_{ROQ} /ROI_{EOQ}
25670	44946	9,8	2,4	0,6	4,1

Table 4: Profit and ROI for the ROQ model compared to profit and ROI for the EOQ model.

As one may mention, the profit in case of ROQ model is less than in case of EOQ, but the ROI is bigger. That's quite predictable result. The most important is the magnitudes, actually. The model is useful in the case if ROI grows up faster than profit falls down. In this example profit is around 40% less and the ROI is approximately 4 times bigger.

6. Conclusion and further research

Notwithstanding the EOQ and ROQ models include the same parameters and variables, they have some differences.

One of the main outcomes, derived from the using of ROI as an objective for inventory control is a decentralization of decision making. The main difference from the EOQ model can be formulated as follows: goods in inventory should not be managed independently of each other and be ordered in quantities obtained from simple models.

In this thesis the ROI criterion for EOQ model was considered, with the additional assumption that the firm has no capital investment apart the inventory. As it was already said, see for example, Dan Trietsch (1995), the ROI criterion should be used only if we take into the consideration the company-wide approach. This can be proved if we consider the objective functions for both of the models (EOQ and ROQ). The classical EOQ model considers the cost minimization as an objective and the ROQ model considers the ROI maximization as an objective. If to compare these two objectives one may mention that cost is an absolute measure of the firm's efficiency and the ROI is a relative one. As far as relative values for single items are not additive, the model uses derivation of order quantities for all goods. That's why model require company-wide approach.

If to go back to the cost of capital in the both of the models, one may mention that the way this costs are determined is different for both of the models. In EOQ model the opportunity cost of capital estimated according to some external factors, but in the ROQ model the cost of the capital appears in the optimization process. In practice it might be not fully correct. For example, in the EOQ model this cost is not actually external, especially if one considers the whole firm at the same time the ROQ model is affected by some influences from the market and other environmental factors.

There are a lot of possibilities for further research. For example, the model can be extended using some other constraints, such as capacity limitation. On the other hand one may apply this model to some special cases, such as backorders or discounts.

Appendix I

In order to find formulas for budget constraint and Lagrange multiplier, let's consider function:

$$L(Q_i,\lambda)=\sum_{i=1}^{n}\frac{A_i d_i}{Q_i}+\frac{1}{2}\sum_{i=1}^{n}Q_i v_i r+\lambda\left(\frac{1}{2}\sum_{i=1}^{n}Q_i v_i-B\right) \qquad (1)$$

$$\frac{\partial L(Q_i,\lambda)}{\partial Q_i}=-\frac{A_i d_i}{Q^2{}_i}+\frac{1}{2}v_i r+\frac{1}{2}\lambda v_i \qquad (2)$$

$$\frac{\partial L(Q_i,\lambda)}{\partial \lambda}=\frac{1}{2}\sum_{i=1}^{n}Q_i v_i-B \qquad (3)$$

From (2):

$$Q_i=\sqrt{\frac{2A_i d_i}{v_i(\lambda+r)}} \qquad (4)$$

From (3):

$$\frac{1}{2}\sum_{i=1}^{n}Q_i v_i=B \qquad (5)$$

(4)→(5)

$$\frac{1}{2}\sum_{i=1}^{n}\sqrt{\frac{2A_i d_i}{v_i(\lambda+r)}}v_i=B \qquad (6)$$

$$\sqrt{r+\lambda}=\frac{\sum_{i=1}^{n}\sqrt{A_i d_i v_i}}{B\sqrt{2}} \qquad (7)$$

(7)→(4) and if $A_i=A_j$

$$(Q_i)_B=\sqrt{\frac{d_i}{v_i}}\frac{2B}{\sum_{i=1}^{n}\sqrt{d_i v_i}} \qquad (8)$$

If $A_i \neq A_j$

$$\lambda=\frac{1}{4B^2}\left(\sum_{i=1}^{n}\sqrt{2A_i d_i v_i}\right)^2-r \qquad (9)$$

$$(Q_i)_B = \sqrt{\frac{A_i d_i}{v_i}} \frac{2B}{\sum_{i=1}^{n} \sqrt{A_i d_i v_i}} = \left[(Q_i)_{HW} = \sqrt{\frac{2A_i d_i}{v_i r}} \right] = \frac{B\sqrt{2r}}{\sum_{i=1}^{n} \sqrt{A_i d_i v_i}} (Q_i)_{HW} = \frac{B}{\sum_{i=1}^{n} \sqrt{\frac{A_i d_i v_i}{2r}}} (Q_i)_{HW} =$$

$$= \frac{B}{\frac{1}{2} \sum_{i=1}^{n} v_i \sqrt{\frac{2A_i d_i}{v_i r}}} (Q_i)_{HW} \left[= \sqrt{\frac{2A_i d_i}{v_i r}} = (Q_i)_{HW} \right] = \frac{B}{\frac{1}{2} \sum_{i=1}^{n} v_i (Q_i)_{HW}} (Q_i)_{HW} \qquad (8')$$

According to the definition we have

$$\frac{1}{2} \sum_{i=1}^{n} v_i (Q_i)_{HW} = B_{HW} \qquad (10)$$

$$(Q_i)_B = \frac{B}{B_{HW}} (Q_i)_{HW} \qquad (11)$$

New formula for λ may be obtained now from (9) and (10)

$$\lambda = \frac{1}{4B^2} \left(\sum_{i=1}^{n} \sqrt{2A_i d_i v_i} \right)^2 - r = \frac{1}{(2B)^2} \left(\sum_{i=1}^{n} \sqrt{2A_i d_i v_i} \right)^2 - r = \left[\sum_{i=1}^{n} \sqrt{2A_i d_i v_i} = B_{HW} 2\sqrt{r} \right] =$$

$$= \frac{1}{(2B)^2} \left(B_{HW} 2\sqrt{r} \right)^2 - r = \frac{(B_{HW} 2)^2 r}{(2B)^2} - r = \frac{(B_{HW})^2 r}{(B)^2} - r = \left[\left(\frac{B_{HW}}{B} \right)^2 - 1 \right] r \qquad (12)$$

Appendix II

In order to find optimal budget constraint and optimal order quantity corresponding to this constraint, let's consider the function:

$$ROI((Q_i)_B)=\frac{\sum_{i=1}^{n}(p_i-v_i)d_i-\Phi-(\sum_{i=1}^{n}\frac{A_i d_i}{(Q_i)_B}+\frac{1}{2}\sum_{i=1}^{n}(Q_i)_B v_i r)}{\frac{1}{2}B}$$

$$(Q_i)_B=\frac{B}{B_{HW}}(Q_i)_{HW}$$

$$ROI((Q_i)_B)=\frac{\sum_{i=1}^{n}(p_i-v_i)d_i-\Phi-(\sum_{i=1}^{n}\frac{A_i d_i}{\frac{B}{B_{HW}}(Q_i)_{HW}}+\frac{1}{2}\sum_{i=1}^{n}\frac{B}{B_{HW}}(Q_i)_{HW} v_i r)}{\frac{1}{2}B}$$

$$ROI((Q_i)_B)=\mathrm{f(B)}$$

$$\mathrm{f(B)}=\frac{\sum_{i=1}^{n}(p_i-v_i)d_i}{\frac{1}{2}B}-\frac{\Phi}{\frac{1}{2}B}-\frac{\sum_{i=1}^{n}\frac{A_i d_i}{\frac{B}{B_{HW}}(Q_i)_{HW}}}{\frac{1}{2}B}-\frac{\frac{1}{2}\sum_{i=1}^{n}\frac{B}{B_{HW}}(Q_i)_{HW} v_i r}{\frac{1}{2}B}=$$

$$=\frac{2\sum_{i=1}^{n}(p_i-v_i)d_i}{B}-\frac{2\Phi}{B}-\frac{\sum_{i=1}^{n}\frac{\frac{2A_i d_i}{(Q_i)_{HW}}}{B_{HW}}}{B^2}-\sum_{i=1}^{n}\frac{(Q_i)_{HW} v_i r}{B_{HW}}$$

$$\frac{\partial \mathrm{f(B)}}{\partial B}=\frac{-2\sum_{i=1}^{n}(p_i-v_i)d_i}{B^2}+\frac{2\Phi}{B^2}+4\frac{\sum_{i=1}^{n}\frac{A_i d_i B_{HW}}{(Q_i)_{HW}}}{B^3}$$

$$\frac{\partial^2 \mathrm{f(B)}}{\partial B^2}=\frac{4\sum_{i=1}^{n}(p_i-v_i)d_i}{B^3}-\frac{4\Phi}{B^3}-12\frac{\sum_{i=1}^{n}\frac{A_i d_i B_{HW}}{(Q_i)_{HW}}}{B^4}$$

$$\frac{\partial \mathrm{f(B)}}{\partial B}=0$$

$$\sum_{i=1}^{n}(p_i-v_i)d_i-\Phi-2\frac{\sum_{i=1}^{n}\frac{A_i d_i B_{HW}}{(Q_i)_{HW}}}{B}=0 \quad (\mathrm{NB!B}\neq 0)$$

$$\frac{\sum_{i=1}^{n}\frac{2A_i d_i B_{HW}}{(Q_i)_{HW}}}{B}=\sum_{i=1}^{n}(p_i-v_i)d_i-\Phi$$

$$B_{\max} = \frac{2\sum_{i=1}^{n} \frac{A_i d_i B_{HW}}{(Q_i)_{HW}}}{\sum_{i=1}^{n}(p_i - v_i)d_i - \Phi}$$

Now we need to investigate the $\frac{\partial^2 \mathrm{f(B)}}{\partial B^2}$ in order to know if the extreme value exists in the point

$$\frac{\partial^2 \mathrm{f(B)}}{\partial B^2} = \frac{4\sum_{i=1}^{n}(p_i - v_i)d_i}{B^3} - \frac{4\Phi}{B^3} - 12\frac{\sum_{i=1}^{n} \frac{A_i d_i B_{HW}}{(Q_i)_{HW}}}{B^4} =$$

$$= \frac{4B(\sum_{i=1}^{n}(p_i - v_i)d_i - \Phi) - 12\sum_{i=1}^{n} \frac{A_i d_i B_{HW}}{(Q_i)_{HW}}}{B^4}$$

It should be true that

$\frac{\partial^2 \mathrm{f(B)}}{\partial B^2} \neq 0$ for the $B_{\max} = \frac{2\sum_{i=1}^{n} \frac{A_i d_i B_{HW}}{(Q_i)_{HW}}}{\sum_{i=1}^{n}(p_i - v_i)d_i - \Phi}$

(for existence of extreme value for *f(B)* at B_{max}) and

$\frac{\partial^2 \mathrm{f(B)}}{\partial B^2} < 0$ for $B_{\max} = \frac{2\sum_{i=1}^{n} \frac{A_i d_i B_{HW}}{(Q_i)_{HW}}}{\sum_{i=1}^{n}(p_i - v_i)d_i - \Phi}$

(in order to have the maximum value for *f(B)* at B_{max})

$$\frac{\partial^2 \mathrm{f(B)}}{\partial B^2} = \frac{4\frac{2\sum_{i=1}^{n} \frac{A_i d_i B_{HW}}{(Q_i)_{HW}}}{\sum_{i=1}^{n}(p_i - v_i)d_i - \Phi}(\sum_{i=1}^{n}(p_i - v_i)d_i - \Phi) - 12\sum_{i=1}^{n} \frac{A_i d_i B_{HW}}{(Q_i)_{HW}}}{\left[\frac{2\sum_{i=1}^{n} \frac{A_i d_i B_{HW}}{(Q_i)_{HW}}}{\sum_{i=1}^{n}(p_i - v_i)d_i - \Phi}\right]^4} = \frac{8\sum_{i=1}^{n} \frac{A_i d_i B_{HW}}{(Q_i)_{HW}} - 12\sum_{i=1}^{n} \frac{A_i d_i B_{HW}}{(Q_i)_{HW}}}{\left[\frac{2\sum_{i=1}^{n} \frac{A_i d_i B_{HW}}{(Q_i)_{HW}}}{\sum_{i=1}^{n}(p_i - v_i)d_i - \Phi}\right]^4} =$$

$$= \frac{(8-12)\sum_{i=1}^{n} \frac{A_i d_i B_{HW}}{(Q_i)_{HW}}}{\left[\frac{2\sum_{i=1}^{n} \frac{A_i d_i B_{HW}}{(Q_i)_{HW}}}{\sum_{i=1}^{n}(p_i - v_i)d_i - \Phi}\right]^4} = \left[\sum_{i=1}^{n} \frac{A_i d_i B_{HW}}{(Q_i)_{HW}} \neq 0\right] = \frac{(8-12)\sum_{i=1}^{n} \frac{A_i d_i B_{HW}}{(Q_i)_{HW}}}{\left[\frac{16\left(\sum_{i=1}^{n} \frac{A_i d_i B_{HW}}{(Q_i)_{HW}}\right)^4}{\left(\sum_{i=1}^{n}(p_i - v_i)d_i - \Phi\right)^4}\right]} = \frac{(8-12)}{\frac{16\left(\sum_{i=1}^{n} \frac{A_i d_i B_{HW}}{(Q_i)_{HW}}\right)^3}{\left(\sum_{i=1}^{n}(p_i - v_i)d_i - \Phi\right)^4}}$$

$$\frac{\left(\sum_{i=1}^{n}\frac{A_i d_i B_{HW}}{(Q_i)_{HW}}\right)^3}{\left(\sum_{i=1}^{n}(p_i - v_i)d_i - \Phi\right)^4} > 0 \quad \rightarrow \quad \frac{\partial^2 \mathrm{f(B)}}{\partial B^2} < 0$$

Let's find a corresponding Q for B_{max}

$$(Q_i)_{B_{\max}} = \frac{2\sqrt{2r}\, B_{HW}(Q_i)_{HW} \sum_{i=1}^{n}\frac{A_i d_i}{(Q_i)_{HW}}}{\sum_{i=1}^{n}\sqrt{A_i d_i v_i}\sum_{i=1}^{n}(p_i - v_i)d_i - \Phi}$$

$$= \frac{\frac{2}{\sqrt{r}}\sqrt{\frac{A_i d_i}{v_i}}\left(\sum_{i=1}^{n}\sqrt{A_i d_i v_i}\right)}{\sum_{i=1}^{n}(p_i - v_i)d_i - \Phi} = 2\sqrt{\frac{A_i d_i}{v_i r}}\,\frac{\sum_{i=1}^{n}\sqrt{A_i d_i v_i}}{\sum_{i=1}^{n}(p_i - v_i)d_i - \Phi}$$

Appendix III

Budget constraints and shadow price in terms of TC_{HW} and H:

$$B_{HW} = \frac{1}{2}\sum_{i=1}^{n}(Q_i)_{HW} v_i = \frac{1}{2}\sum_{i=1}^{n}\sqrt{\frac{2A_i d_i}{v_i r}}v_i = \frac{1}{2r}\sum_{i=1}^{n}\sqrt{2A_i d_i v_i r} = \left[TC_{HW} = \sum_{i=1}^{n}\sqrt{2A_i d_i v_i r}\right] = \frac{1}{2r}TC_{HW}$$

$$B_{\max} = \frac{2\sum_{i=1}^{n}\frac{A_i d_i B_{HW}}{(Q_i)_{hw}}}{\sum_{i=1}^{n}(p_i - v_i)d_i - \Phi} = \frac{2\sum_{i=1}^{n}\frac{A_i d_i \frac{1}{2}\sum_{i=1}^{n} v_i (Q_i)_{hw}}{(Q_i)_{hw}}}{\sum_{i=1}^{n}(p_i - v_i)d_i - \Phi} = \left[H = \sum_{i=1}^{n}(p_i - v_i)d_i - \Phi\right] =$$

$$= \frac{B_{HW}\sum_{i=1}^{n}\frac{2A_i d_i}{(Q_i)_{HW}}}{H} = \frac{B_{HW} r\sum_{i=1}^{n}\frac{2A_i d_i}{v_i r (Q_i)_{HW}} v_i}{H} = \frac{B_{HW} r\sum_{i=1}^{n}\frac{(Q_i)^2{}_{HW}}{(Q_i)_{HW}} v_i}{H} =$$

$$= \frac{2B_{HW} r \frac{1}{2}\sum_{i=1}^{n}(Q_i)_{HW} v_i}{H} = \frac{2rB^2{}_{HW}}{H} = \left[B_{HW} = \frac{1}{2r}TC_{HW}\right] = \frac{1}{2rH}TC^2{}_{HW}$$

→

$$\lambda = \left[\left(\frac{B_{hw}}{B}\right)^2 - 1\right]r = \left[\left(\frac{H}{TC_{HW}}\right)^2 - 1\right]r$$

Reference

Arcelus, F. J., & Srinivasan, G.. (1985). A ROI-Maximizing EOQ Model Under Variable Demand and Markup Rates. Engineering Costs and Production Economics, 9(1-3), 113. Retrieved November 23, 2011, from ABI/INFORM Global.

Arcelus, F. J., & Srinivasan, Inventory Policies Under Various Optimizing Criteria and Variable Markup RatesManagement Science 1987 33:756-762;

Axsäter, Sven. (2006) Inventory control, New York, Springer.

Biederman, David. "Reversing Inventory Management" Traffic World (12 December 2004): 1.

Chamberlain, Neil W.: The firm: micro-economic planning and action. (New York, McGraw-Hill, 1962)

Dan Trietsch, Revisiting ROQ: EOQ for Maximizing Company-Wide Return on Investment, Journal of Operational Research Society, 46, 1995, 507-515.

Edward A. Silver, David F. Pyke, Rein Peterson, Inventory management and production planning and scheduling.(New York: Wiley, 1998)

Eilon, Samuel, (1960) Economic Batch-Size Determination for Multi-Product Scheduling, Operational Research Quarterly, Vol. 11, No. 1/2 (Mar. - Jun., 1960), pp. 77-79 Stable URL: http://www.jstor.org/stable/3007182

Eilon, Samuel. (1964). Dragons in pursuit of the EBQ. *Opl Res. Q.* **15**, 347-354. (A response to Tate)

F. E. Raymond (1931) *Quantity and Economy in Manufacture*. McGraw-Hill, New York.

G. M. Fikhtengol'ts, The Fundamentals of Mathematical Analysis. London ; New York : Pergamon Press, 1965.

Hadley, G. And Whitin, T., Analysis of Inventory Systems, Prentice Hall, Englewood Cliffs, NJ 1963

Halskau, Ø and Thorstenson, A, (1998) The EOQ and the ROQ and Their Relations to the ROI (Extended Version), Working Paper, Norwegian School of Logistics.

Hax, Arnoldo C. and Candea, Dan (1984) Production and inventory management. New Jersey, Prentice-Hall.

Johnson, Lynwood A. and Montgomery, Douglas C. (1974) England: John Wiley & Sons.

Kaplan, R.S., (1982) Advanced Management Accounting, Prientice-Hall, Eneglewood Cliffs, N.J.

Kotler, P., 1971. Marketing Decision Making: A ModelBuilding Approach. Holt-Rinehart and Winston, New York.

Meredith, Jack R. and Shafer, Scott M. (2006) Operations Management for MBAs, England: John Wiley & Sons. ISBN-13: 978-0471351429

Morse, W.J. and Schneider, J.H. (1979) “Cost Minimization, Return on Investment, Residual Income: Alternative Criteria for Inventory Models.” Accounting and Business Res., 9,3.

Otake, T. and Min, K.J. and Cheng-Kang Chen. 1999. Inventory and investment in setup operations under return on investment maximization. Comput. Oper. Res. 26, 9 (August 1999), 883-899.

Otake, T. and Min, K.J., Inventory and investment in quality improvement under return on investment maximization. Computers and Operations Research, **28** (2001), pp. 113–124.

Rosenberg, David. (1991). Optimal Price-Inventory Decisions: Profit vs. ROII. IIE Transactions, 23 (1), 17. Retrieved November 23, 2011, from ABI/INFORM Global.

Schroeder, R. G. and Krishnan, R. (1976), RETURN ON INVESTMENT AS A CRITERION FOR INVENTORY MODELS. Decision Sciences, 7: 697–704.

Tate, T. B. (1965) In defence of the economic batch quantity. (Letter to the editor). *Opl Res. Q.* **16**, 109-112.

Tersine, Richard J. (1994) Principles of inventory and materials management. 4th edition. New Jersey, Prentice-Hall.

Van der Veen, B. (1967) *Introduction to the theory of operational research*. New York, Springer.

Waters, C. D. J. (2003) Logistics: an introduction to supply chain management, Palgrave Macmillan.

Waters, C. D. J. (1992) Inventory Control and Management, England: John Wiley & Sons. ISBN13: 9780471930815

Whitin, T.M., (1969) Inventory Control and Price Theory, Management Science, 2, 1

Willard Zangwill. 1987. From EOQ towards ZI. Manage. Sci. 33, 10 (October 1987), 1209-1223. DOI=10.1287/mnsc.33.10.1209